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A Study on Two-Phase Service Queueing System with Server Vacations

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Abstract
This paper mainly deals with the analysis of queueing system in which customers require services in two stages are common. Two-phase queueing systems are gaining importance due to their wide variety of applications. Batch mode service in the first phase followed by individual service in the second phase. Because of the mixed mode of services, these kinds of two phase queues do not fit into the usual network of queues as a particular case. The need for this kind of modeling has been greatly discussed in this paper
Keywords: Sojourn time, busy cycle, service cycle, idle time, service time, system size

I. Introduction

Krishna and Lee (1990) have studied a two phase Markovian queueing system. Customers arrive according to poisson process and service times are exponentially distributed. Doshi (1991) has generalized the model studied in Krishna and Lee (1990). In this, the service times in both the first and second phases are assumed to follow general distributions. Madan(2001) studied two similar types of vacation models for M/G/1 Queueing system. Shahkar and Badamchi (2008) have studied the two phase queueing system with Bernoulli server vacation .Gautam Choudhury (2011) has generalized the model with delaying repair . Jau-Chuan Ke and Wen Lea Pearn (2011) have studied the analysis of the multi-server system with a modified Bernoulli vacation schedule.

II. Methodology

A queueing system with two phase service, customers arrive at the system according to a poisson process with arrival rate λ. In the first phase, the server operates on an exhaustive batch mod service in waiting customers and the arrivals which occur during the batch service. Thus, this batch consists of the waiting customers at the commencement of the batch service and also those customers who arrive at the system during the batch service on completion of the batch mode service, the same server takes the entire batch of customers to the second phase for individual service following. First in first out queue discipline. The customers who arrive while the server is busy in the second phase, wait in a queue at the first phase for the next batch mode service.

After completing the second phase of individual services exhaustively to the batch, the server returns to the first phase to start the next batch mode service. If some customers are present at the first phase, he/she starts the batch mode service immediately. On the other hand if the queue is empty, he/she immediately leaves for a vacation of random duration and returns to the system at the end of the vacation. If there is at least one customer waiting in the first phase, he/she immediately starts a batch service, but if the system is empty, he takes the next vacation and so on.

III. Model Description

Batch service times are independent and identically distributed random variables following an arbitrary function B(t), $t > 0$ and the individual service times are i.i.d. with an arbitrary distribution function F(t), $t > 0$ and vacation times are i.i.d. according to an arbitrary distribution function V(t), $t > 0$.

Let the random variables B, S and V denoted batch service times, individual service times and vacation times respectively. Assume that the system is in steady state and

$\rho = \lambda E(S) < 1$

$\gamma = \lambda E(B)$

3.1 Busy Cycle

The time interval from the commencement of the batch service at the end of a vacation to the commencement of the next batch service after availing at least one vacation is a busy cycle.

3.2 Service Cycle

The time interval between two consecutive batch services without any vacation is a service cycle.

Notations

N – system size at the beginning of a batch service
N_1 – system size at the end of the batch service
N_2 – system size at the end of individual services to the customers of the batch service
X – number of customers arrivals during a batch service
Y – number of customers arrivals ≥ 1 during a vacation
P_{20} – P (the system is empty at the end of the second phase

The random variables N, N_1, N_2, X and Y are interrelated.

$N_1 = N + X$

N_2 = number of customers arrivals during the N1 individual services

$$N = \begin{cases} N_2 & \text{if } N_2 > 0 \\ Y & \text{if } N_2 = 0 \end{cases}$$

Probability generating function of B(t), F(t) and V(t)

$$\overline{B}(\theta) = \int_0^\infty e^{-\theta t} dB(t)$$

$$\overline{F}(\theta) = \int_0^\infty e^{-\theta t} dF(t)$$

$$\overline{V}(\theta) = \int_0^\infty e^{-\theta t} dV(t)$$

a_K = P{K poisson arrivals during a time interval having a function A(t)}

$$= \int_0^\infty \frac{e^{-\lambda t}(\lambda t)^K}{K!} dA(t)$$

$$E(z^K) = \sum_{k=0}^\infty z^K \int_0^\infty \frac{e^{-\lambda t}(\lambda t)^K}{K!} dA(t)$$

$$= \int_0^\infty e^{-\lambda t} \sum_{k=0}^\infty \frac{(\lambda zt)^K}{K!} dA(t)$$

$$= \int_0^\infty e^{-\lambda t} e^{\lambda tz} dA(t)$$

$$= \int_0^\infty e^{-\lambda t + \lambda tz} dA(t)$$

$$= \int_0^\infty e^{-t(\lambda - \lambda z)} dA(t)$$

$$= \overline{A}(\lambda - \lambda z) \qquad (1)$$

where $\overline{A}(0)$ is the LST of A(t).

$$a_0 = P\,(\text{No arrival}) = \overline{A}(\lambda)$$

$P^*(z)$, $P_1^*(z)$ and $P_2^*(z)$ denote the PGF's of N, N_1 and N_2 respectively.

$$\begin{aligned} P_1^*(z) &= E(z^{N_1}) \\ &= E(z^{N+X}) \\ &= E(z^N.z^X) \\ &= E(z^N).E(z^X). \end{aligned}$$

arrival and service are independent

$$= P^*(z)\overline{B}(\lambda - \lambda z) \tag{2}$$

Let Xi denote the number of arrivals during the i-th individual service for i = 1, 2, 3, …, N.

$$\begin{aligned} P_2^*(z) &= E(z^{N_2}) \\ &= E[E(z^{N_2}/N_1 \supset K)] \\ &= \sum_{k=1}^{\infty} E(z^{X_1+X_2+\cdots+X_K})P(N_1 = K) \\ &= \sum_{k=1}^{\infty} (E(z^{X_1}))^K P(N_1 = K) \quad \text{since } X_i \text{ are independent} \\ &= \sum_{k=1}^{\infty} (\overline{F}(\lambda - \lambda z))^K P(N_1 = K) \end{aligned}$$

$$= P_1^*(\overline{F}(\lambda - \lambda z)) \tag{3}$$

Since Y denote the number of customer arrivals ≥ 1 during a vacation.

$$\begin{aligned} P(Y = k) &= \sum_{i=1}^{\infty} \{(P(Y = k\ /\ k \text{ customers arrive during the } i^{th} \text{ vacation})\} \\ &= \sum_{i=1}^{\infty} b_0^{i-1} b_k \\ &= \frac{b_k}{1 - b_0} \\ &= \frac{b_k}{1 - \overline{V}(\lambda)} \end{aligned}$$

$$\begin{aligned} P^*(z) &= E(z^N) \\ &= \sum_{k=1}^{\infty} z^k P(N = k) \end{aligned}$$

But $P(N = k) = P(N_2 = k)$ with proba 1 when $N_2 > 0$
$= P(Y = k)$ with proba P_{20} when $N_2 = 0$

$$\begin{aligned} P^*(z) &= \sum_{k=1}^{\infty} z^k P(N_2 = k) + P_{20} \sum_{k=1}^{\infty} z^k P(Y = k) \\ &= (P(z) - P_{20}) + \frac{P_{20}(\overline{V}(\lambda - \lambda z) - \overline{V}(\lambda))}{1 - \overline{V}(\lambda)} \end{aligned}$$

$$= P_2^*(z) - P_{20}\left\{\frac{1-\overline{V}(\lambda-\lambda z)-\overline{V}(\lambda)}{1-\overline{V}(\lambda)}\right\} \tag{4}$$

Combine (2), (3) and (4) we get

$$P^*(z) = P^*(\overline{F}(\lambda-\lambda z)\overline{B}(\lambda-\lambda\overline{F}(\lambda-\lambda z)) - P_{20}\left\{1-\frac{(\overline{V}(\lambda-\lambda z)-\overline{V}(\lambda)}{1-\overline{V}(\lambda)}\right\} \tag{5}$$

Let

$$\begin{aligned}
g^{(0)}(z) &= z \\
g^{(1)}(z) &= g(z) \\
&= \overline{F}(\lambda-\lambda z) \\
&\vdots \\
g^n(z) &= g(g^{n-1}(z)) \\
&= g^{n-1}(g(z)) \qquad n \ge 1 \\
h^{(1)}(z) &= h(z) \\
&= \overline{B}(\lambda-\lambda z) \\
L(z) &= 1-\frac{(\overline{V}(\lambda-\lambda z)-\overline{V}(\lambda))}{1-\overline{V}(\lambda)}
\end{aligned}$$

For $\rho < 1$, $z_\infty = 1$ is the unique solution of the equation $z = \overline{F}(\lambda-\lambda z)$ inside the region $|z| \le 1$. $g^{(n)}(z)$ approaches $z = 1$ as $n \to \infty$ and $L(g^{(n)}(z))$ approaches $L(1) = 0$ using $\overline{V}(\lambda-\lambda z) \to V(0) = 1$.

The quantity P_{20} can be obtained from the equation (5).

$$P^*(z) = P^*(g^{(1)}(z))h(g^{(1)}(z)) - P_{20}L(z)$$

$$P^*(z) = \prod_{i=1}^{\infty} h(g^{(1)}(z)) - P_{20}\sum_{k=0}^{\infty}\left\{\prod_{i=1}^{k} h(g^{(1)}(z))L(g^{(k)}(z))\right\}$$

Since the number of customers at the beginning of a batch service is strictly positive.

$$P^*(0) = P(N=0) = 0$$

$$P^*(\overline{F}(\lambda)\ \ \overline{B}(\lambda-\lambda\overline{F}(\lambda)) - P_{20} = 0$$

Since $g(0) = \overline{F}(\lambda)$

$$\begin{aligned}
P_{20} &= P^*(\overline{F}(\lambda)\ \ \overline{B}(\lambda-\lambda\overline{F}(\lambda)) \\
&= P^*(g(0)\ \ \overline{B}(\lambda-\lambda\overline{F}(\lambda)) \\
&= h(g(0))P^*(g(0)) \\
&= h(g(0))\left\{\prod_{i=1}^{\infty} h(g^{i+1}(0)) - P_{20}\sum_{k=0}^{\infty}\left[\prod_{i=1}^{k} h(g^{i+1}(0)\right]L(g^{k+1}(0))\right\}
\end{aligned}$$

$$P_{20} = \frac{h(g(0))\prod_{i=1}^{\infty} h(g^{i+1}(0))}{1 + h(g(0))\sum_{k=0}^{\infty}\left[\prod_{i=1}^{k} h(g^{i+1}(0)\right]L(g^{k+1}(0))}$$

$$= \frac{\prod_{i=1}^{\infty} h(g^{(i)}(0))}{1 + \sum_{k=0}^{\infty} L(g^{k+1}(0))\prod_{i=1}^{k+1} h(g^{k+1}(0)}$$

3.3 The Probability Generating Function of System Size

Let M denote the system size when a random customer leaves the system after his service completion and $M^*(z)$ be the PGF of M.

Let $N_1 = n_1$ and $K = k$ where n_1 is the size of the batch to which the random customer belongs and k is his position in that batch, then by using length biased sampling technique

$$P(K = k/N_1 = n_1) = \left(\frac{1}{n_1}\right) n_1 P(N_1 = n_1)/E(N_1)$$

$$= P(N_1 = n_1)/E(N_1)$$

The number of customers in the system when a random customer departs is the sum of $(n_1 - k)$ customers in the batch and the number of new arrivals that occur during the K service completions.

$$E(N_1) = [\gamma(1 - \overline{V}(\lambda) + P_{20}\lambda E(V)]/(1 - \rho)(1 - \overline{V}(\lambda)]$$

$V(t) = 1$, $t > 0$ when the server does not take any vacation.
$B(t) = 1$, $t > 0$ there is no batch service.

Mean and Variance

$$E(w_s) = E(S) + \frac{\lambda}{1-\rho}\frac{E(s^2)}{2} + \frac{E(B^2)}{2E(N_1)} - \frac{E^2(B)}{E(N_1)} + \frac{E(B)}{\lambda}\frac{P_{20}E(V^2)}{2(1-\overline{V}(\lambda)E(N_1)}$$

$$Var(w_s) = E(w_s^2) - (E(w_s))^2.$$

3.4 Server Utilization

Let T1 and T2 denote the durations of time the server is busy in the system. Let $\overline{T}_1(\theta)$ and $\overline{T}_2(\theta)$ denote the LSTs of T_1 and T_2 respectively.

$$\overline{T}_1(\theta) = \sum_{r=1}^{\infty}[\overline{B}(\theta)P_1^*(\overline{F}(\theta))]^r(1 - P_{20})^{r-1}P_{20}$$

$$\overline{T}_2(\theta) = \sum_{r=1}^{\infty}(\overline{V}(\theta))^r(\overline{V}(\lambda))^r(1 - \overline{V}(\lambda))$$

$$E(T_1) = \frac{[E(B) + E(N_1)E(s)]}{P_{20}}$$

$$E(T_2) = \frac{E(V)}{1 - \overline{V}(\lambda)}$$

The long run proportion is given by

$$u = \frac{E(T_1)}{E(T_1) + E(T_2)}$$

$$u = \frac{\left(\dfrac{E(B)+E(N_1)E(s)}{P_{20}}\right)}{\dfrac{E(B)+E(N_1)E(s)}{P_{20}} + \dfrac{E(V)}{1-\overline{V}(\lambda)}}$$

$$= \frac{E(B)+E(N_1)E(s)}{P_{20}} \times \frac{P_{20}(1-\overline{V}(\lambda))}{(E(B)+E(N_1)E(s))}(1-\overline{V}(\lambda)) + P_{20}E(V)$$

$$u = \frac{(E(B) + E(N_1)E(s))(1-\overline{V}(\lambda))}{(E(B) + E(N_1)E(s))(1-\overline{V}(\lambda)) + P_{20}E(V)}$$

IV. Table

Case (i) $\lambda = 5$, E(B) = 1/15

$\frac{1}{E(V)}$	$E(w_s)$
5	0.56
4	0.58
3	0.6
2	0.71
1	1.18

Case (ii) $\lambda = 5$, E(B) = 1/10

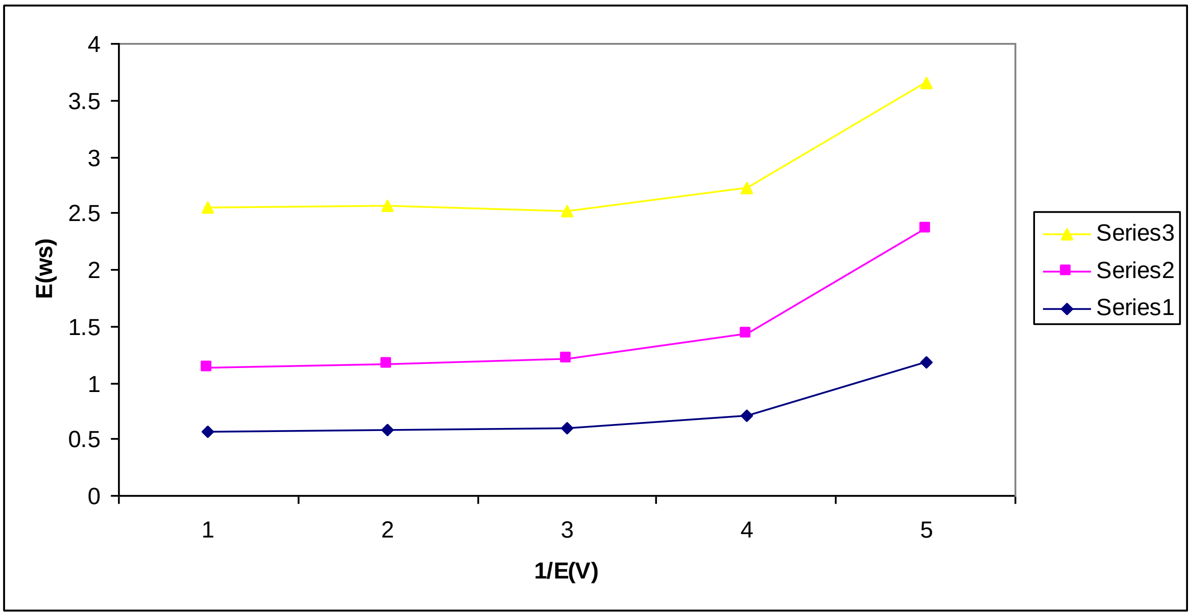
$\frac{1}{E(V)}$	$E(w_s)$
5	0.58
4	0.59
3	0.61
2	0.72
1	1.18

Case (iii) $\lambda = 5$, E(B) = 1/2

$\frac{1}{E(V)}$	$E(w_s)$
5	1.41
4	1.4
3	1.31
2	1.3
1	1.3

Graph

$\lambda = 5$



From the above graph indicates the mean system time decreases with the decrease in the mean vacation time .

V. Conclusion

In this paper we studied the vacation model of a queuing system in which customers are served in two phases. Server vacations are introduced to utilize the idle time of the server so that idle times are used to perform some useful jobs other than the service to the customers in the system . In fact , such jobs may even include a simple break for taking some rest in serving in some other queue. The study of vacation models of queuing systems help the effects of server vacations on the system performance .

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